

Gleichungen mit Polynomdivision lösen

1 Aufgaben

Figure 1:

Gegeben sind die Polynome f und eine oder mehrere zugehörige Nullstellen x_k ($k = 1, 2, \dots$). Man ermittle sämtliche reellen Nullstellen von f .

- i) $f(x) = x^3 - 2x^2 - 2x + 4$; $x_1 = 2$
 ii) $f(x) = x^4 - 6x^3 + 3x^2 + 26x - 24$; $x_1 = 3$; $x_2 = -2$
 iii) $f(x) = x^3 - 2x + 1$; $x_1 = 1$
 iv) $f(x) = 2x^4 - 3x^3 - 10x^2 + 5x - 6$; $x_1 = -2$; $x_2 = 3$

2 Lösungen

```
(%i1) kill(all);
(%o0) done
```

2.1 Teilaufgabe (i)

Figure 2:

i) $f(x) = x^3 - 2x^2 - 2x + 4$; $x_1 = 2$

```
(%i1) f(x) := x**3 - 2*x**2 - 2*x + 4;
(%o1) f(x) := x^3 - 2x^2 + (-2)x + 4
```

direkte Lösung mit `realroots()` oder `solve()`

```
(%i2) l:realroots(f(x));
(%o2) [x = -47453133/33554432, x = 47453133/33554432, x = 2]
```

```
(%i3) l:solve(f(x)=0,x);
(%o3) [x = -sqrt(2), x = sqrt(2), x = 2]
```

durch Polynomdivision auf eine quadratische Gleichung reduzieren

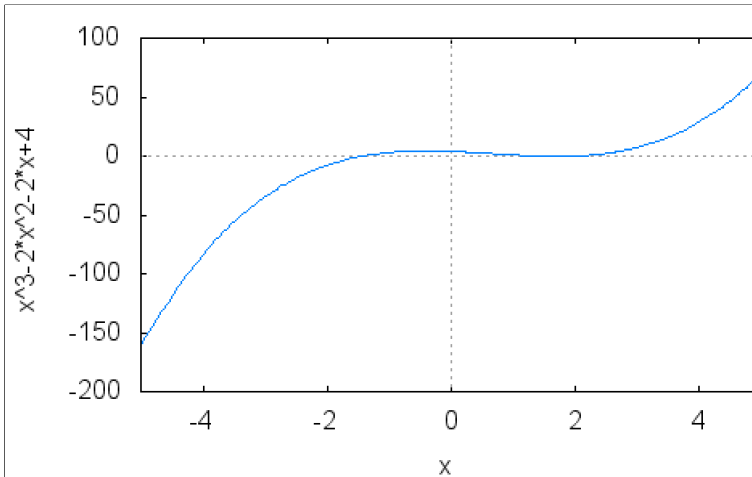
```
(%i4) l:divide(f(x),x-2);
(%o4) [x^2 - 2, 0]
```

```
(%i5) f:l[1];
(%o5) x^2 - 2
```

```
(%i6) l:solve(f,x);
(%o6) [x = -sqrt(2), x = sqrt(2)]
```

```
(%i7) wxplot2d([f(x)], [x,-5,5])$
```

```
(%t7)
```



```
(%i8) kill(all);
```

```
(%o0) done
```

2.2 Teilaufgabe (ii)

Figure 3:

ii) $f(x) = x^4 - 6x^3 + 3x^2 + 26x - 24$; $x_1 = 3$; $x_2 = -2$

```
(%i1) f(x):=x**4-6*x**3+3*x**2+26*x-24;
```

```
(%o1) f(x):=x4-6 x3+3 x2+26 x-24
```

```
(%i2) l:divide(f(x),(x-3)*(x+2));
```

```
(%o2) [x2-5 x+4, 0]
```

```
(%i3) f:l[1];
```

```
(%o3) x2-5 x+4
```

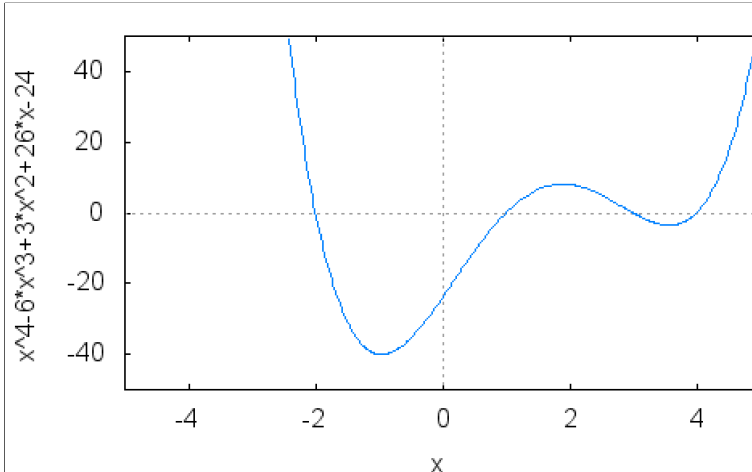
```
(%i4) l:solve(f,x);
```

```
(%o4) [x=1, x=4]
```

```
(%i5) wxplot2d([f(x)], [x,-5,5],[y,-50,50])$
```

plot2d: some values were clipped.

```
(%t5)
```



```
(%i6) kill(all);
(%o0) done
```

2.3 Teilaufgabe (iii)

Figure 4:

```
iii)  $f(x) = x^3 - 2x + 1$ ;  $x_1 = 1$ 
```

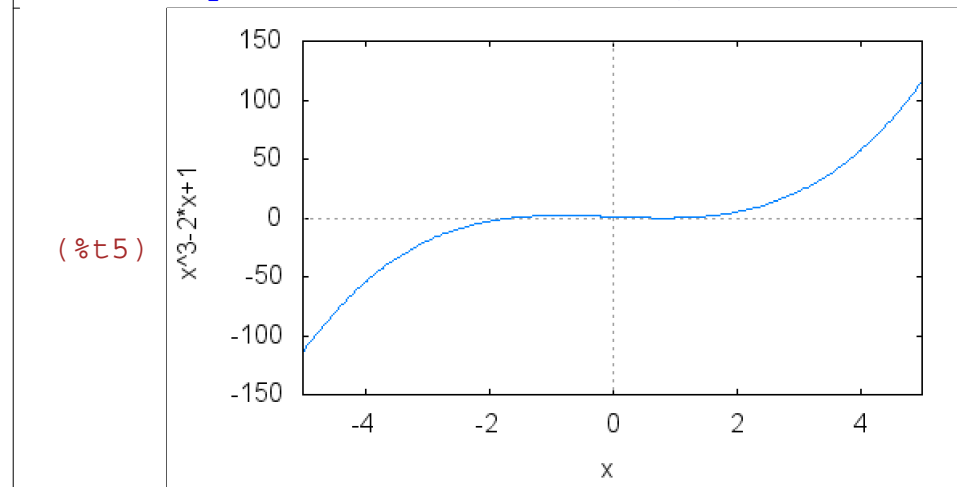
```
(%i1) f(x):=x**3-2*x+1;
(%o1) f(x):=x3-2 x+1
```

```
(%i2) l:divide(f(x),x-1);
(%o2) [ $x^2+x-1$ , 0]
```

```
(%i3) f:l[1];
(%o3)  $x^2+x-1$ 
```

```
(%i4) l:solve(f,x);
(%o4) [ $x = -\frac{\sqrt{5}+1}{2}$ ,  $x = \frac{\sqrt{5}-1}{2}$ ]
```

```
(%i5) wxplot2d([f(x)], [x,-5,5])$
```



```
(%i6) kill(all);
(%o0) done
```

2.4 Teilaufgabe (iv)

Figure 5:

```
iv)  $f(x) = 2x^4 - 3x^3 - 10x^2 + 5x - 6$ ;  $x_1 = -2$ ;  $x_2 = 3$ 
```

```
(%i1) f(x):=2*x**4-3*x**3-10*x**2+5*x-6;
(%o1) f(x):=2 x4-3 x3+(-10)x2+5 x-6
```

```
(%i2) l:divide(f(x),(x+2)*(x-3));
(%o2) [ $2x^2-x+1$ , 0]
```

```
(%i3) f:l[1];  
(%o3) 2 x2-x+1  
  
(%i4) l:solve(f,x) /* es gibt nur noch komplexe Lösungen */;  
(%o4) [x=- $\frac{\sqrt{7} \%i-1}{4}$ , x= $\frac{\sqrt{7} \%i+1}{4}$ ]  
  
(%i5) wxplot2d([f(x)], [x,-5,5],[y,-40,20])$  
plot2d: some values were clipped.
```

